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Spatial Correlations in Compressible Granular Flows

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Abstract

For a freely evolving granular fluid, the buildup of spatial correlations in density and flow field is described using fluctuating hydrodynamics. The theory for incompressible flows is extended to the general, compressible case, including longitudinal velocity and density fluctuations, and yields qualitatively different results for long range correlations. The structure factor of density fluctuations shows a maximum at finite wavenumber, shifting in time to smaller wavenumbers and corresponding to a growing correlation length. It agrees well with two-dimensional molecular dynamics simulations.

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In most studies of *rapid granular flows*, also called the *granular gas* regime [1], the inelasticity of granular collisions is assumed to be the most important feature that distinguishes these flows from usual liquid or gas flows. The dynamics is modeled by a single *inelasticity parameter* $\epsilon = 1 - \alpha^2$, where α is the coefficient of normal restitution. As a consequence a granular flow can only be maintained in driven systems, where energy is put into the system e.g. by gravity, shear or in vibrated layers [1]. Also quite some work has been done on the freely evolving granular fluid [2–6], which has been shown to be *linearly unstable* (onset of clustering instability) with respect to spatial fluctuations in density, $\delta n(\mathbf{r}, t) = n(\mathbf{r}, t) - \langle n \rangle$ [2]. In Ref. [7], an analytic description has been given of the buildup of equal time spatial correlations in the flow field,

$$G_{\alpha\beta}(\mathbf{r}, t) = \frac{1}{V} \int d\mathbf{r}' \langle u_\alpha(\mathbf{r} + \mathbf{r}', t) u_\beta(\mathbf{r}', t) \rangle, \tag{1}$$

of a system initialized in a spatially homogeneous state. The theory is based on fluctuating hydrodynamics and the assumption of incompressible flow, $\boldsymbol{\nabla} \cdot \mathbf{u} = 0$. This theory yields predictions, including long range tails $\sim r^{-d}$, that for nearly elastic particles ($\epsilon \lesssim 0.2$) agree well with 2-D molecular dynamic simulations up to large distances. Here we will extend the theory to the general, compressible case, allowing us to calculate longitudinal velocity and density fluctuations. In fact, the structure factor corresponding to density fluctuations, $S_{nn}(k, t) = V^{-1} \langle \delta n(\mathbf{k}, t) \delta n(-\mathbf{k}, t) \rangle$, has been analyzed before by Deltour and Barrat [4]. The difference between compressible and incompressible flow is best appreciated in Fourier space, where velocity correlations are described by the tensor $S_{\alpha\beta}(\mathbf{k}, t)$. Both $G_{\alpha\beta}(\mathbf{r}, t)$ and its Fourier transform $S_{\alpha\beta}(\mathbf{k}, t) = V^{-1} \langle u_\alpha(\mathbf{k}, t) u_\beta(-\mathbf{k}, t) \rangle$ are isotropic tensors and can be decomposed into two scalar isotropic functions in the following way:

$$\begin{aligned} G_{\alpha\beta}(\mathbf{r}, t) &= \hat{r}_\alpha \hat{r}_\beta G_\parallel(r, t) + (\delta_{\alpha\beta} - \hat{r}_\alpha \hat{r}_\beta) G_\perp(r, t) \\ S_{\alpha\beta}(\mathbf{k}, t) &= \hat{k}_\alpha \hat{k}_\beta S_\parallel(k, t) + (\delta_{\alpha\beta} - \hat{k}_\alpha \hat{k}_\beta) S_\perp(k, t), \end{aligned} \tag{2}$$

where carets denote unit vectors. In a system of *elastic* hard spheres (EHS) for times larger than the mean free time t_0, the correlation functions are given by the equilibrium values, i.e. $G_{\alpha\beta}(\mathbf{r}, t) = [T/mn] \delta_{\alpha\beta} \delta(\mathbf{r})$, containing self-correlations only, and $G_{nn}(r, t) = n\delta(\mathbf{r}) + n^2[g(r) - 1]$, where $g(r)$ is the pair distribution function in thermal equilibrium. For convenience we substract self-correlations and introduce the functions $G^+_{\alpha\beta}(\mathbf{r}, t) \equiv G_{\alpha\beta}(\mathbf{r}, t) - [T(t)/mn] \delta_{\alpha\beta} \delta(\mathbf{r})$ and $S^+_{\alpha\beta}(\mathbf{k}, t) \equiv S_{\alpha\beta}(\mathbf{k}, t) - [T(t)/mn] \delta_{\alpha\beta}$. Note that $T(t)$ is measured in energy units ($k_B = 1$). The structure factor of transverse velocity fluctuations, $S^+_\perp(k, t)$, was calculated analytically in Ref. [7] and shown to yield a long range r^{-d} tail in $G_\perp(r, t)$ and $G_\parallel(r, t)$ in case the fluctuations in the flow field are incompressible, i.e. $S^+_\parallel(k, t) = 0$.

In this Letter the structure factors $S_{\alpha\beta}(\mathbf{k}, t)$ and $S_{nn}(k, t)$ and corresponding spatial correlation functions $G_{\alpha\beta}(\mathbf{r}, t)$ and $G_{nn}(r, t)$ will be calculated and compared with 2-D molecular dynamics simulations for inelastic hard disk systems. We show in particular by explicit calculation that for small inelasticity ($\epsilon \lesssim 0.2$) $S^+_\parallel(k, t)$ is essentially vanishing for all wavenumbers except at very small k values ($k \lesssim 1/\xi_\parallel$), where the assumption of incompressible $\mathbf{u}$ fluctuations, made in Ref. [7], breaks down. Consequently, the most important role of $S^+_\parallel(k, t)$ is to provide an exponential cutoff for the r^{-d} tail at the largest scales $r \gtrsim 2\pi\xi_\parallel$. At larger inelasticities the contributions from $S^+_\parallel(k, t)$ modify $G_\parallel(r, t)$ and $G_\perp(r, t)$ significantly at all distances.

The hydrodynamic equations for the unforced inelastic hard sphere (IHS) fluid possess an exact solution, the *homogeneous cooling state* (HCS), with a homogeneous density n and temperature $T(t)$, and vanishing flow field. Energy is dissipated at a rate $2\gamma_0\omega T$, where $\gamma_0 = \epsilon/2d$ and where the collision frequency $\omega = \Omega_d\chi(n)n\sigma^{d-1}\sqrt{T/\pi m}$ is calculated from the Enskog-Boltzmann equation [8] for a dense system of hard disks or spheres ($d = 2, 3$). Here $\Omega_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface area of a d-dimensional unit sphere, σ and m the sphere diameter and mass, both of which are set equal to unity, and $\chi(n)$ the pair correlation function at contact. For detailed definitions and derivations we refer to [2]. In the following, we will assume that IHS hydrodynamics can be described by the standard hydrodynamic equations supplemented by an energy sink term, which is evaluated in the local homogeneous cooling state. The equations of change for the macroscopic fields then become

$$\begin{aligned}
&\partial_t n + \boldsymbol{\nabla}\cdot(n\mathbf{u}) = 0\\
&\partial_t \mathbf{u} + \mathbf{u}\cdot\boldsymbol{\nabla}\mathbf{u} = -\frac{1}{n}\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}\\
&\partial_t T + \mathbf{u}\cdot\boldsymbol{\nabla}T = -\frac{2}{dn}(\boldsymbol{\nabla}\cdot\mathbf{J} + \boldsymbol{\Pi} : \boldsymbol{\nabla}\mathbf{u}) - 2\gamma_0\omega[n,T]T,
\end{aligned} \tag{3}$$

where all fields and fluxes depend on $(\mathbf{r}, t)$. A possible justification of these equations to lowest order in ϵ can be found in Ref. [9], as well as a discussion of higher order terms. The pressure tensor $\boldsymbol{\Pi} = \boldsymbol{\Pi}_0 + \boldsymbol{\Pi}_1$ is given by $\boldsymbol{\Pi}_0 = p\mathbf{I} = nT(1+\Omega_d\chi n\sigma^d/2d)\mathbf{I}$, with $\mathbf{I}$ the identity matrix, and $\boldsymbol{\Pi}_1 = -2\eta\{\boldsymbol{\nabla}\mathbf{u}\}_s - \zeta(\boldsymbol{\nabla}\cdot\mathbf{u})\mathbf{I}$, with $\{\boldsymbol{\nabla}\mathbf{u}\}_{s,\alpha\beta} = \frac{1}{2}(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - 2\delta_{\alpha\beta}\boldsymbol{\nabla}\cdot\mathbf{u}/d)$, and the heat flow $\mathbf{J} = -\kappa\boldsymbol{\nabla}T$. Here η, ζ and κ are, respectively, the shear viscosity, bulk viscosity and heat conductivity, given by the Enskog theory for EHS with temperature $T(t)$ still depending explicitly on time to account for the homogeneous cooling [7]. The equations of change for the mesoscopic fields are obtained from the above equations by adding fluctuating terms to the pressure tensor and heat flow, denoted by $\hat{\boldsymbol{\Pi}}$ and $\hat{\mathbf{J}}$ respectively [10]. They are characterized by a vanishing average and correlations which are local in space and time, the strength of which is assumed to be determined by the standard fluctuation-dissipation theorem:

$$\begin{aligned}
\langle\hat{\Pi}_{\alpha\beta}(\mathbf{r},t)\hat{\Pi}_{\gamma\delta}(\mathbf{r}',t')\rangle &= 2T[\eta(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma})\\
&\quad + (\zeta - \frac{2}{d}\eta)\delta_{\alpha\beta}\delta_{\gamma\delta}]\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\\
\langle\hat{J}_\alpha(\mathbf{r},t)\hat{J}_\beta(\mathbf{r}',t')\rangle &= 2\kappa T^2\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),
\end{aligned} \tag{4}$$

with transport coefficients depending on $T(t)$. We are interested in the buildup of correlations between spatial fluctuations in a system that is prepared in a homogeneous state at an initial temperature T_0 and reaches the HCS within a few mean free times $t_0 = 1/\omega[n, T_0]$. Therefore, we can linearize the above equations around a homogeneous density n and a temperature $T(t) = T_0/[1 + \gamma_0 t/t_0]^2$, and a vanishing flow field. At this point it is convenient to make the change of variables, $d\tau = \omega[n, T(t)]dt$, where τ is the average number of collisions a particle has suffered within a time t, $\delta n(\mathbf{r}, t) = n\delta\nu(\mathbf{r}, \tau)$, $\mathbf{u}(\mathbf{r}, t) = \sqrt{T(t)}\mathbf{w}(\mathbf{r}, \tau)$, $\delta T(\mathbf{r}, t) = T(t)\delta\theta(\mathbf{r}, \tau)$, $\hat{\boldsymbol{\Pi}}(\mathbf{r}, t) = n\omega[n, T(t)]\sqrt{T(t)}\hat{\boldsymbol{\pi}}(\mathbf{r}, \tau)$ and $\hat{\mathbf{J}}(\mathbf{r}, t) = n\omega[n, T(t)]T(t)\hat{\boldsymbol{\jmath}}(\mathbf{r}, \tau)$. In these new variables the noise strengths of the reduced fluctuating pressure tensor $\hat{\boldsymbol{\pi}}$ and heat flow $\hat{\boldsymbol{\jmath}}$ are *time independent*, and the equations

of change for the mesoscopic Fourier modes $\delta\nu(\mathbf{k},\tau)$, $\mathbf{w}(\mathbf{k},\tau)$ and $\delta\theta(\mathbf{k},\tau)$ become ordinary differential equations with *time independent* coefficients (valid for $kl_0 \lesssim 1$ where $l_0 = \sqrt{2T(t)}/\omega[n,T(t)]$ is the time independent mean free path):

$$\begin{aligned}
\frac{\partial \delta\nu}{\partial \tau} &= -\frac{ikl_0}{\sqrt{2}} w_l \\
\frac{\partial w_{\perp\alpha}}{\partial \tau} &= \gamma_0(1-k^2\xi_\perp^2)w_{\perp\alpha} - ik\hat{\pi}_{\alpha l} \\
\frac{\partial w_l}{\partial \tau} &= \gamma_0(1-k^2\xi_l^2)w_l - \frac{ikl_0}{\sqrt{2}}\left(\frac{p}{nT}\right)\delta\theta \\
&\quad -\frac{ikl_0}{\sqrt{2}}\left(\frac{1}{nT\chi_T}\right)\delta\nu - ik\hat{\pi}_{ll} \\
\frac{\partial \delta\theta}{\partial \tau} &= -\gamma_0(1+k^2\xi_T^2)\delta\theta - \frac{ikl_0}{\sqrt{2}}\left(\frac{2p}{dnT}\right)w_l \\
&\quad -2\gamma_0(1+\frac{n}{\chi}\frac{\partial\chi}{\partial n})\delta\nu - ik\frac{2}{d}\hat{\jmath}_l.
\end{aligned} \tag{5}$$

Here we have introduced the time independent correlation lengths $\xi_\perp$, ξ_l and ξ_T, defined by $\xi_\perp^2 = \nu/\omega\gamma_0$ with $\nu = \eta/mn$ the kinematic viscosity, $\xi_l^2 = [2\nu(d-1)/d + \zeta/mn]/\omega\gamma_0$ and $\xi_T^2 = 2\kappa/dn\omega\gamma_0$, and the isothermal compressibility $\chi_T = (\partial n/\partial p)_T/n$. The subscript α in the equation for $\mathbf{w}_\perp$ refers to any of the $(d-1)$ directions perpendicular to $\mathbf{k}$, and the subscript l denotes the longitudinal direction along $\mathbf{k}$. To calculate the structure factors we also need the Fourier modes with $\mathbf{k}$ replaced by $-\mathbf{k}$.

Since the transverse velocity $\mathbf{w}_\perp$ is decoupled from the other modes, its structure factor $S_\perp(k,t) = \langle u_{\perp\alpha}(\mathbf{k},t)u_{\perp\alpha}(-\mathbf{k},t)\rangle/V$ can be obtained in the analytic form [7]

$$S_\perp(k,t) = \frac{T(t)}{n}\left\{1 + \frac{\exp[2\gamma_0\tau(1-k^2\xi_\perp^2)]-1}{1-k^2\xi_\perp^2}\right\}, \tag{6}$$

which is valid for $kl_0 \lesssim 1$. The same result has been obtained from a more microscopic approach, using *ring kinetic theory* [11].

The density, longitudinal velocity and temperature modes are coupled and their equations of change can be written in matrix representation as

$$\frac{\partial}{\partial\tau}\boldsymbol{\psi}(\mathbf{k}) = \mathbf{M}(\mathbf{k})\boldsymbol{\psi}(\mathbf{k}) + \hat{\mathbf{f}}(\mathbf{k}), \tag{7}$$

where $\boldsymbol{\psi}$ is the column vector with components $\psi_1 = \delta\nu$, $\psi_2 = w_l$ and $\psi_3 = \delta\theta$, and the hydrodynamic matrix $\mathbf{M}$ and the noise vector $\hat{\mathbf{f}}$ are given by Eqs. (5). Note that the elements $M_{31}(\mathbf{k})$ and $M_{33}(\mathbf{k})$, entering the temperature equation, depend on the energy dissipation term. In this notation the equal time correlations obey the equation of change

$$\begin{aligned}
&\frac{\partial}{\partial\tau}\langle\psi_\alpha(\mathbf{k},\tau)\psi_\beta(-\mathbf{k},\tau)\rangle = M_{\alpha\gamma}(\mathbf{k})\langle\psi_\gamma(\mathbf{k},\tau)\psi_\beta(-\mathbf{k},\tau)\rangle \\
&+M_{\beta\gamma}(-\mathbf{k})\langle\psi_\alpha(\mathbf{k},\tau)\psi_\gamma(-\mathbf{k},\tau)\rangle + C_{\alpha\beta}(k),
\end{aligned} \tag{8}$$

where $\alpha,\beta,\ldots = 1,2,3$ label the components $\delta\nu$, w_l and $\delta\theta$. These equations constitute a set of 3×3 linear ordinary differential equations, of which only 6 are independent. The matrix of

noise strengths $C_{\alpha\beta}(k)$, defined through $\langle \hat{f}_\alpha(\mathbf{k},\tau)\hat{f}_\beta(-\mathbf{k},\tau')\rangle = C_{\alpha\beta}(k)\delta(\tau-\tau')$, has only two nonvanishing components, namely $C_{22} = 2V\gamma_0 k^2\xi_l^2/n$ and $C_{33} = 4V\gamma_0 k^2 \xi_T^2/dn$. We have solved the above set of equations numerically, starting from initial equilibrium correlations, of which the only nonvanishing ones are $\langle\psi_1(\mathbf{k},0)\psi_1(-\mathbf{k},0)\rangle = VT\chi_T$, $\langle\psi_2(\mathbf{k},0)\psi_2(-\mathbf{k},0)\rangle = V/n$ and $\langle\psi_3(\mathbf{k},0)\psi_3(-\mathbf{k},0)\rangle = 2V/dn$ (for $k \neq 0$). The most important new results with respect to Ref. [7] are the structure factors $S_\parallel(k,t)$ and $S_{nn}(k,t)$, and the correlation function $G_{nn}(r,t)$. In Fig. 1, we show the results for these structure factors, including $S_\perp(k,t)$, for a system with area fraction $\phi = 0.245$ and $\alpha = 0.9$ together with the results from a molecular dynamics simulation of 50000 inelastic hard disks. We observe that $S_\parallel(k\to 0,t) = S_\perp(k \to 0,t)$, implying for large distances an asymptotic behavior $G_{\alpha\beta}(\mathbf{r},t) \sim S_\perp(k\to 0,t)\delta_{\alpha\beta}\delta(\mathbf{r})$, and thus the absence of algebraic long range correlations on the largest scales ($r \gg 2\pi\xi_\parallel$). Therefore, we can already conclude that the asymptotic behavior of $G_\perp(r,t)$ and $G_\parallel(r,t)$ cannot be r^{-d}. Instead the r^{-d} tail obtained in Ref. [7] describes intermediate behavior which is exponentially cut off at a distance determined by the width of $S^+_\parallel(k,t)$. This width can be estimated from the eigenvalues of the hydrodynamic matrix, more precisely from the dispersion relation of the heat mode [3], which is a pure longitudinal velocity w_l for $k \to 0$. To second order in k its dispersion relation is given by $z_H(k) = \gamma_0(1-k^2\xi_\parallel^2)$, with

$$\xi_\parallel^2 = \xi_l^2 + \frac{l_0^2}{2\gamma_0^2}\left[\frac{1}{nT\chi_T} - \frac{p}{nT}\left(1 + \frac{n}{\chi}\frac{\partial\chi}{\partial n} - \frac{p}{dnT}\right)\right]. \tag{9}$$

Note that $\xi_\parallel \sim 1/\epsilon$ for small inelasticity, whereas $\xi_\perp \sim \xi_l \sim \xi_T \sim 1/\sqrt{\epsilon}$ [see Fig. 2(c)]. To a good approximation $S_\parallel(k,t)$ for small wavenumbers is given by expression (6) with $\xi_\perp$ replaced by $\xi_\parallel$. This approximation is excellent up to wavenumbers where the exact numerical result for $S_\parallel(k,t)$ shows a little dip (see Fig. 1, $\tau = 19.4$, $k \simeq 0.1$). At about the same wavenumber the structure factor $S_{nn}(k,t)$ reaches its maximal value, which grows in time. The exact position of this maximum shifts in time to smaller wavenumbers corresponding to a growing correlation length. This can be explained by the following argument: for $k \to 0$ density fluctuations $\delta n(\mathbf{k},t)$ are decoupled from the heat mode and we expect that $S_{nn}(k\to 0,t)$ remains at its initial equilibrium value; at small, but nonvanishing k, $\delta n(\mathbf{k},t)$ couples in $\mathcal{O}(k)$ to the unstable heat mode and the maximum of $k\exp[2z_H(k)\tau]$ shifts in time to smaller wavenumbers.

The estimate for $S_{nn}(k,t) \simeq S_{nn}(k,0)\exp[2z_H(k)\tau]$, used in Ref. [4], differs in two aspects from our predictions: (i) it neglects the wavenumber dependence of the coupling of density fluctuations to the heat mode, giving for $S_{nn}(k,t)$ a decreasing function of k, and therefore cannot explain the growing correlation length; (ii) it neglects the fluctuating parts of the pressure tensor and heat flow, causing only numerical deviations from our prediction. Note that seven out of the eight sets of data points shown in Fig. 9 of Ref. [4] are in the crossover or nonlinear time regime, which is estimated in Ref. [7] to occur at $\tau_{\rm cr} \simeq 65$ for $\alpha = 0.9$ and $\phi = 0.4$ and where our linear theory breaks down.

Using the above approximation for $S_\parallel(k,t)$, the structure factor $S^+_{\alpha\beta}(\mathbf{k},t)$ can be written as

$$\begin{aligned} S^+_{\alpha\beta}(\mathbf{k},t) \approx \frac{T(t)}{n}\int_0^s \mathrm{d}s' \exp(s')\Big[&\hat{k}_\alpha\hat{k}_\beta \exp(-s'k^2\xi_\parallel^2) \\ &+(\delta_{\alpha\beta} - \hat{k}_\alpha\hat{k}_\beta)\exp(-s'k^2\xi_\perp^2)\Big], \end{aligned} \tag{10}$$

where $s = 2\gamma_0\tau$. If the system is thermodynamically large ($L \gg 2\pi\xi_\parallel$), $G^+_\parallel(r,t) = \hat{r}_\alpha\hat{r}_\beta G^+_{\alpha\beta}(\mathbf{r},t)$ and $G^+_\perp(r,t) = (\delta_{\alpha\beta} - \hat{r}_\alpha\hat{r}_\beta)G^+_{\alpha\beta}(\mathbf{r},t)/(d-1)$ can be obtained by performing integrals over $\mathbf{k}$ space; the resulting $G^+_\parallel(r,t)$ and $G^+_\perp(r,t)$ can then be expressed as integrals over simple functions. Here we only quote the results for $d = 2$. Using

$$\int \frac{d\mathbf{q}}{(2\pi)^2} \sin^2\theta e^{i\mathbf{q}\cdot\mathbf{x} - sq^2} = \frac{1}{2\pi x^2}[1 - \exp(-x^2/4s)], \tag{11}$$

where $\cos\theta = \hat{\mathbf{q}}\cdot\hat{\mathbf{x}}$, we obtain

$$\begin{aligned} G^+_\lambda(r,t) \approx \frac{T(t)}{n} \Bigg(&\frac{1}{4\pi\xi_\lambda^2}\int_0^s ds' \frac{\exp(s' - x_\lambda^2/4s')}{s'} \\ &+ \frac{m_\lambda}{2\pi r^2}\int_0^s ds' e^{s'}\left[\exp\left(-\frac{x_\parallel^2}{4s'}\right) - \exp\left(-\frac{x_\perp^2}{4s'}\right)\right]\Bigg) \end{aligned} \tag{12}$$

for $\lambda = \parallel, \perp$, where $x_\lambda = r/\xi_\lambda$, $m_\parallel = 1$ and $m_\perp = -1$. The approximation of *incompressible* fluid flow of Ref. [7] is obtained in the limit $\xi_\parallel \to \infty$. It is consistent with the thermodynamic concept of incompressibility, i.e. $\chi_T = 0$ in (9), implying an infinite speed of sound. Fig. 2 shows $g_\parallel(x_\perp, s) = [n\xi_\perp^2/T(t)]G^+_\parallel(r,t)$ and $g_\perp(x_\perp, s) = [n\xi_\perp^2/T(t)]G^+_\perp(r,t)$ in the above approximation for different ratios $\xi_\parallel/\xi_\perp$ at $s = 2$. In order to see r^{-d} behavior the second term of (12) should dominate over the first. For $\xi_\parallel^2 \gg \xi_\perp^2$, $\exp(-x_\perp^2/4s')$ can be neglected with respect to $\exp(-x_\parallel^2/4s')$, and the second term behaves algebraically if $x_\parallel^2 \lesssim 4s$. Restricting ourselves to $1 \lesssim s \lesssim 10$ (i.e. moderate times where the correlations have grown above noise level), we can estimate that the range of algebraic decay is restricted to $r \lesssim 2\pi\xi_\parallel$, where the factor 2π is chosen for convenience. For $r \gtrsim 2\pi\xi_\parallel$, the remaining exponent cuts off the r^{-d} tail.

The predicted spatial velocity correlations $G_\parallel(r,t)$ and $G_\perp(r,t)$ have been obtained by numerically performing inverse Bessel transformations on the numerical results for $S_\parallel(k,t)$ and $S_\perp(k,t)$. The result for $G_\parallel(r,t)$ corresponding to Fig. 1 includes an intermediate r^{-2} tail and is shown in Fig. 3(a). Fig. 3(b) shows the corresponding spatial density correlation $G_{nn}(r,t)$ obtained numerically from $S_{nn}(k,t)$. It confirms that the present theory correctly predicts the buildup of density correlations, including a negative correlation centered around a distance which grows in time as $\sqrt{\tau}$.

At small inelasticity ($\epsilon \lesssim 0.2$) the functions $G_\parallel(r,t)$ and $G_\perp(r,t)$, calculated here from the full set of hydrodynamic equations, differ for $r \lesssim 2\pi\xi_\parallel$ only slightly from the results for incompressible flow fields (see discussion in Ref. [7]). However, the algebraic tails $\sim r^{-d}$ in $G_\parallel(r,t)$ and $G_\perp(r,t)$, derived in Ref. [7] for $r \gtrsim 2\pi\xi_\perp$, are exponentially cut off for $r \gtrsim 2\pi\xi_\parallel$. As the correlation lengths $\xi_\perp \sim 1/\sqrt{\epsilon}$ and $\xi_\parallel \sim 1/\epsilon$ are well separated for small ϵ [see Fig. 2(c)], there is an intermediate range of r values where the algebraic tail $\sim r^{-d}$ in $G_\parallel(r,t)$ can be observed.

At higher inelasticity $\xi_\parallel$ and $\xi_\perp$ are not well separated and, as a consequence, there does not exist a spatial regime in which the longitudinal fluctuations in the flow field can be neglected and the regime of validity of the incompressible theory of Ref. [7] has shrunk to zero. Fig. 3(c) compares results from incompressible and compressible fluctuating hydrodynamics with simulation data for $G_\perp(r,t)$ at $\alpha = 0.6$ and $\phi = 0.4$ and confirms the validity of

the compressible fluctuating hydrodynamics description up to reasonably large inelasticities. Note that $G_{nn}(r,t)$ can only be calculated from the compressible theory.

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FIGURES

FIG. 1. Theoretical predictions (solid lines) for $S_\perp(k,t)$, $S_\parallel(k,t)$ and $S_{nn}(k,t)$ versus $k\sigma$ for $\phi = 0.245$ ($l_0 \simeq 0.8$) and $\alpha = 0.9$, where $\xi_\perp = 4$ and $\xi_\parallel = 17$, at $\tau = 19.4$, 40 and 48.4, compared with results from a single molecular dynamics run of 50000 particles, implying a minimal wavenumber $k_{\rm min}\sigma = 2\pi\sigma/L \simeq 0.016$. All structure superimposed on the plateau values presents long range correlations of dynamic origin; equilibrium structure in S_{nn} is only present for $k\sigma \gtrsim 2\pi$.

FIG. 2. (a) ${}^{10}\log g_\parallel(x_\perp, s=2)$ versus ${}^{10}\log x_\perp$ from incompressible fluctuating hydrodynamics (solid line) and present theory (12) (dashed lines from left to right: $\xi_\parallel/\xi_\perp = 1,2,5,10$); as $\xi_\parallel/\xi_\perp$ decreases, the r^{-2} tail is cut off exponentially at smaller distances and finally disappears at $\xi_\parallel = \xi_\perp$. (b) $g_\perp(x_\perp, s=2)$ versus $x_\perp = r/\xi_\perp$; the depth of the minimum decreases with decreasing $\xi_\parallel/\xi_\perp$ and finally disappears at $\xi_\parallel = \xi_\perp$. (c) $\xi_\parallel/\xi_\perp$ versus area fraction ϕ.

FIG. 3. (a) ${}^{10}\log[|G_\parallel|/T]$ versus ${}^{10}\log r$. (b) G_{nn} versus r; the same parameters α, ϕ, τ as in Fig. 1 are used for (a) and (b). (c) $G_\perp/T$ versus r for $\phi = 0.4$ ($l_0 \simeq 0.34$), $\alpha = 0.6$ ($\xi_\perp = 1.46$, $\xi_\parallel = 3.8$) at $\tau = 20$, 40 and 60; in (a) and (c) the solid (dashed) line is the prediction from compressible (incompressible) fluctuating hydrodynamics.

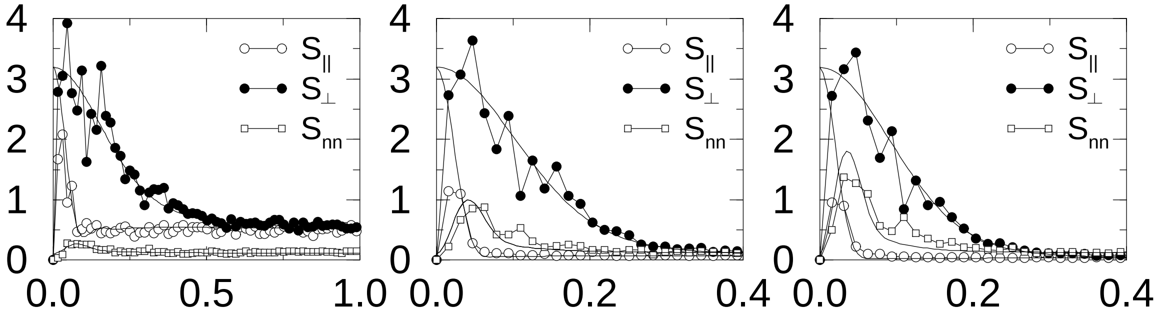

Figure 1

Title: Spatial Correlations in Compressible Flows

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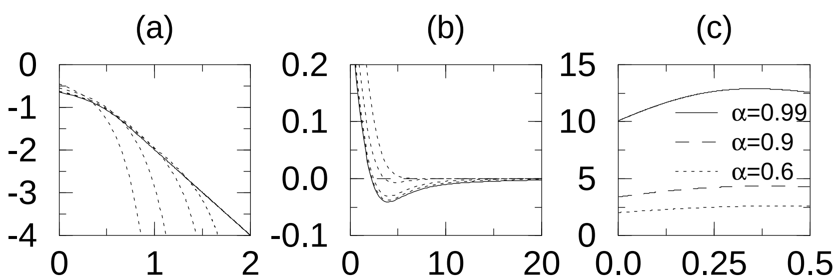

Figure 2

Title: Spatial Correlations in Compressible Flows

Authors: T.P.C. van Noije, M.H. Ernst and R. Brito

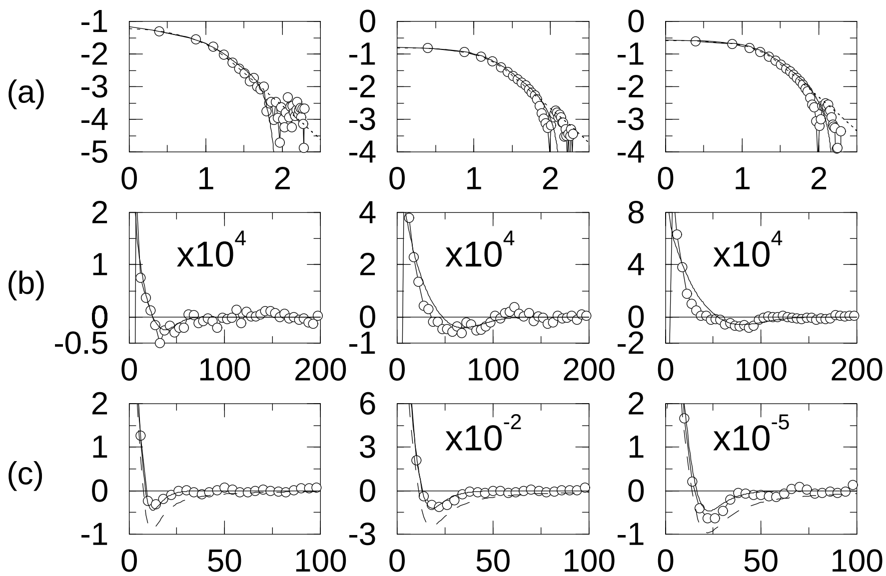

Figure 3

Title: Spatial Correlations in Compressible Flows

Authors: T.P.C. van Noije, M.H. Ernst and R. Brito